Minimax Regret Strategies for Greenhouse Gas Abatement: Methodology and Application*

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Abstract

Classical stochastic programming has already been used with large scale LP models for long-term analysis of energy-environment systems. We propose a *Minimax Regret* formulation suitable for large scale linear programming models. It has been experimentally verified that the minimax regret strategy depends only on the *extremal* scenarios and not on the intermediate ones, thus making the approach computationally efficient. Key results of minimax regret and minimum expected value strategies for Greenhouse Gas abatement in the Province of Québec, are compared.

Keywords: *Energy, Environment, Stochastic Programming, Minimax Regret.*

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1. Introduction

The Global Warming of the terrestrial atmosphere induced by an increase in Greenhouse Gas (GHG) concentration has moved from speculation to fact in the recent years. A major set of studies by the Intergovernmental Panel on Climate Change [12,13,14] summarizes research in these areas over more than a decade.

In spite of the recent Kyoto Protocol establishing abatement targets to be achieved by 2010 in industrialized countries, there still remain large uncertainties, both on the effective attainment of these targets, and on additional reduction levels beyond 2010. The Kyoto Protocol, as well as debates in forums like the IPCC, give some indication of the range in which the long term abatement targets can lie. In a nutshell, the basic problem facing policy makers is to decide upon the levels and timing of emission abatement, climate adaptation and geo-engineering responses to climate change. This directly affects short term energy sector investment decisions, because of the long lead times and even longer life times of projects in electricity generation and transmission, oil refining and natural gas supply. In this paper, we focus on the abatement aspects, which are strongly related to a detailed understanding of the energy system of each country. Specifically, we demonstrate the important impacts that uncertainty may have on policy making, and propose a decision-making paradigm that sheds light on this important issue, namely the Minimax Regret criterion.

The basic question we address in this paper is: *how can we determine an interim energy sector strategy, i.e., before the level and timing of abatement are known with certainty*? The endeavor is to determine a strategy which hedges against all possible requirements to reduce the GHG emissions by various amounts. Although we model the uncertainty in terms of different emission constraints in our model, a similar approach would be equally relevant for a GHG tax situation.

In contrast with the more classical analysis of decision making under risk, where stochastic programming is applied to a set of scenarios, each with a probability of occurrence, we propose for this research the Minimax Regret criterion to determine the hedging strategy under uncertainty. This criterion only needs the list of possible scenarios, without any assumption on their probabilities. In section 2, we describe the formulation of the minimax regret strategy as a linear program. The base model, Extended MARKAL, as well as its Stochastic Programming and Minimax Regret variants, are described in section 3. Section 4 contains the analysis of results, and section 5 concludes the article.

2. Minimax Regret Formulation of a GHG Abatement Model

2.1 A brief reminder of the Minimax Regret Criterion

The Minimax Regret Criterion, also known as Savage Criterion, Raiffa [28], is one of the more credible criteria for selecting decisions under uncertainty, i.e. when the likelihoods of the various possible outcomes are not known with sufficient precision to use the classical expected value or expected utility criteria. In order to fix ideas, we shall assume in what follows that a certain decision problem is couched in terms of a cost to minimize (a symmetric formulation is obtained in the case of a payoff to maximize). Therefore, we may denote by *C(z,s)* the cost incurred when strategy ***s*** is used, and outcome ***z*** occurs. The Regret *R(z,s)* is defined as the difference between the cost incurred with the pair *(z,s)* and the least cost achievable under outcome perfect information on ***z***, i.e.:

$$R(z,s) = C(z,s) - \underset{t\in S}{Min}\, C(z,t), \qquad \forall z \in Z,\ \ s \in S$$

where Z is the set of possible outcomes and S is the set of feasible strategies. Note that, by construction, a regret $R(z,s)$ is always non negative.

A *Minimax Regret (MMR)strategy* is any s^* which minimizes the worst regret, i.e.

$$s^* \in \underset{s\in S}{ArgMin}\{\underset{z\in Z}{Max}\, C(z,s)\}$$

and the corresponding Minimax Regret is equal to:

$$MMR = \underset{s\in S}{Min}\{\underset{z\in Z}{Max}\, C(z,s)\}$$

2.2 Minimax Regret for large scale Linear Programs

We now turn to the application of the above definition to the case when the cost *C(z,s)* is not an explicit expression. Rather, it is implicitly computed via an optimization program. This is the case in particular when using a model such as MARKAL, Fishbone and Abilock [8], Berger et al. [1], Loulou and Lavigne [21], where the long term energy system least cost is computed by solving a large scale linear program with many technical and economic constraints, as well as environmental emission constraints. MARKAL is a 9 period dynamic Linear Program, where each period represents 5 years. In formulation **(1)** below, the *A* matrix defines the very large number of techno-economic constraints of the model, and the last constraint represents the single cumulative greenhouse gas (GHG) emission constraint, i.e. an upper bound on the sum of all emissions over the model's 45 year horizon (the choice of a cumulative emission constraint will be justified in section 3). In the absence of uncertainty, the MARKAL linear program has the following structure:

$$\begin{aligned} M(z) &= \underset{x}{Min}\, c^t x \\ s.t. \quad & Ax \le b \qquad\qquad (1) \\ & e^t x \le z \end{aligned}$$

Where x is the vector of MARKAL decision variables (strategy), and z is the selected cumulative emission target.

The more interesting and more realistic case occurs when the allowed cumulative emission z is uncertain, and will only be known at some future period t^*. Prior to t^*, all decisions are taken under uncertainty, whereas at t^* and later, decisions are taken under perfect knowledge of the cumulative target. It is convenient to decompose the vector x of decision variables into two vectors x_1 and x_2, x_1 representing the decisions to be taken prior to t^*, and x_2 those decisions at t^* and later. We shall assume that the uncertain parameter z may take an arbitrary but finite number n of distinct values: $Z = \{z_1, z_2, \cdots, z_n; z_1 \le z_2 \le \cdots \le z_n\}$

The linear program in **(1)**, which defines an ideal cost *M(z)* that would be incurred under perfect information, allows to define the *Regret* of strategy x as follows:

$$R(x) = c^t x - M(z)$$

Finally, the MMR strategy is an optimal solution to the following optimization program:

$$\begin{aligned} MMR &= \underset{x_1,x_2}{Min}\ \underset{z\in Z}{Max}\left[c_1^t x_1 + c_2^t x_2^z - M(z)\right] \\ s.t. \quad & A_1 x_1 + A_2 x_2^z \ge b, \quad \forall z \in Z \qquad\qquad (2) \\ & e_1^t x_1 + e_2^t x_2^z \le z, \quad \forall z \in Z \end{aligned}$$

The above program is not quite an L.P., but may be converted into one by introducing a new variable:

$$\begin{aligned} MMR &= \underset{x_1,x_2,\phi}{Min}[\phi] \\ s.t. \quad & \phi \ge c_1^t x_1 + c_2^t x_2^z - M(z), \quad \forall z \in Z \\ & A_1 x_1 + A_2 x_2^z \ge b, \qquad \forall z \in Z \qquad\qquad (3) \\ & e_1^t x_1 + e_2^t x_2^z = z, \qquad \forall z \in Z \end{aligned}$$

Note carefully that a *bona fide* strategy x is such that x_1 is common to all outcomes z, whereas there is a different vector x_2^z for each outcome z in Z. This is so because decisions made at t^* and later take into account the knowledge of the true value of z that realizes at t^*. Hence, the LP **(3)** has up to n replications of the constraints, and of the x_2 variables (to be more precise, some constraints which do not involve x_2 variables are not replicated, and therefore, the size of **(3)** may be smaller than n times the size of **(1)**)

Important remark: An unfortunate phenomenon occurs when **(3)** is solved: since all that matters when computing the *MMR* strategy is indeed the value of the Minimax Regret, all other regrets are left free to take any values, as long as these values remain below the *MMR*. This remark is equivalent to saying that **(3)** is highly degenerate. For example, in the instance to be discussed later, the *MMR* is equal to 3,311 M$, but when **(3)** is solved, each of the *n* regrets (i.e. each right-hand-side of the first constraints of **(3)**), is found to be also equal to 3,311 M$. This is undesirable, as in practice, depending upon the actual value of *z* which realizes, the regret can be quite much lower than *MMR*. In order to remove the degeneracy, it is useful to proceed in two phases: first, consider **(3)** as essentially only a way of computing the partial strategy up to the resolution date, i.e. x_1. Next, when this is done, each x_2^z may be computed independently by (i) fixing x_1 at its optimal value (call it x_1^*), and (ii) for each *z*, solving the following *n* linear programs:

$$\begin{aligned} &\underset{x_2}{Min}\left[c_1^t x_1^* + c_2^t x_2 - M(z)\right] \\ &s.t. \quad A_1 x_1^* + A_2 x_2 \geq b \\ &\qquad\ \ e_1^t x_1^* + e_2^t x_2 \leq z \end{aligned} \tag{4}$$

2.3 Computational considerations and a conjecture

The largest LP to solve is clearly **(3)**, which has the same approximate size as a classical stochastic LP defined on the same problem instance, and using the minimum expected value criterion (MEV). In addition, *n-1* smaller problems (4) must be solved, in order to compute the *n-1* non degenerate strategies after date *t**. However, it appears from computational experience (and from intuitive reasoning), that the solution of **(3)** seems to depend only on a restricted problem. This observation is only a conjecture, which we now present, but were unable to prove in any generality.

Conjecture: when solving problem **(2)** *or* **(3)***, it is sufficient to restrict the parameter z to its largest and smallest values* z_1 *and* z_n.

This conjecture was verified in all experiments. If true, it considerably reduces the size of the L.P. **(3)**. The maximum size of **(3)** is approximately twice that of a deterministic problem such as **(1)**, meaning that the number of variables and the number of constraints are both about double those of **(1)**. This has interesting implications in terms of the ease of solving the MMR problem, which thus becomes easier to solve than a classical stochastic LP with more than two outcomes.

The conjecture seems intuitively true in the special case when the uncertain parameter is a single dimensional Right-Hand-Side, since it says (very roughly) that the MMR strategy, once chosen, is always worse for an extreme value of z than it would be for any intermediate value of z. If the uncertainty was

extended to several Right-Hand-Sides, it is not as obvious how the conjecture would be stated, but we offer the following extension: Consider z as a multi-dimensional vector, z_i^j. Index i represents realizations and j represents m different parameters that are uncertain. The uncertain parameters are RHS values of inequality constraints such as emission limits, end-use demands, technology penetration bounds, and resource availability bounds. Assume that for any j, z_1^j represents the RHS value of the most relaxed constraint and z_n^j is the most tight form. We can say that the two 'extreme' scenarios are: $(z_1^1, z_1^2, \ldots, z_1^m)$ and $(z_n^1, z_n^2, \ldots, z_n^m)$ because: $M(\ldots, z_i^j, \ldots) \leq M(\ldots, z_k^j, \ldots) \quad \forall k > i, \forall j$.

Extension of conjecture to multiple uncertain parameters: If m (>1) Right-Hand-Sides are uncertain, it is sufficient to consider the two "extreme" values of the vector z, when solving the MMR problem.

3. Large-Scale Application to Energy-Environment Analysis

3.1 Uncertainty in Energy-Environment Modelling

Many applications of Analysis under risk exist using activity analysis models. We restrict our brief review to recent energy-environment models. A comprehensive description and application of a two-step approach for evaluating an a priori set of hedging energy technologies can be found in Larsson and Wene [15], and Wene [23]. Their method provided for assessing the efficiency and robustness of exogenously determined alternative strategies, by a two-step use of the MARKAL model. Although they did not fully implement the Stochastic Programming paradigm, their work was the initial inspiration to do so by other authors.

Reports on formal inclusion of future uncertainties in long-term energy-environment modeling are scant. Stochastic programming has been used for energy-environment policy modeling recently, but mostly by the very aggregated global models like DICE, Nordhaus [19], MERGE, Manne [18], and CETA-R, Peck and Teisberg [20], all of which have a distinct macroeconomic flavor. While the global models have received wide exposure, they have also been criticized for their inability to faithfully represent the details of national economies. As a consequence, the aggregated economic models must be supplemented by detailed energy bottom-up models such as ours. Fragnière and Haurie [6] and Kanudia and Loulou [13] have used multi-stage stochastic programming formulations within the MARKAL detailed bottom-up model. Another recent work has addressed a similar problem using the two-stage recourse problem formulation, Kanudia [11]. However, the authors are not aware of any application of the minimax regret criterion to a large scale activity analysis model.

3.2 The MARKAL Model

MARKAL [5], [1], is a large scale, technology oriented, activity analysis model, integrating the supply and end-use sectors of an economy, with emphasis on the description of energy related sub-sectors. The model has nine time periods of five years each (thus covering the 45 year span from 1993 to 2037), and utilizes three variables for each technology represented, i.e. the investment, the capacity, and the level of activity of the technology, at each time period. The model uses a discount rate of 5%, which is intended to be representative of the market rate of return on capital. The system's cost includes investment and operations and maintenance costs for all technologies, plus procurement costs for all imported fuels, minus the revenue from exported fuels, minus the salvage value of all residual technologies at the end of the horizon. The model satisfies all important constraints of an energy system, such as conservation of energy flows, satisfaction of demands, conservation of investments, peak-electricity constraints, capacity limits, and many others. In addition, MARKAL allows the optional accounting and/or constraining of emissions of pollutants from all technologies present in the model, by means of emission coefficients and of special constraints, called "emission caps", which may be defined period by period, or in a cumulative fashion. Alternatively, one may impose emission taxes rather than constraints. In order to simultaneously respect these constraints and minimize system cost, MARKAL uses optimization (Linear Programming). A recent modification of MARKAL allows the specification of own price elasticities for all energy services, and therefore will endogenously adjust demands in response to particular scenarios [16].

The database for MARKAL Québec includes more than 500 technologies, approximately 70 energy forms (fuels, plus heat, plus electricity), and 69 categories of energy services, with particular detail in the energy intensive sectors. Full details on the model and data base are available from the authors. Several previous applications of the MARKAL model in Québec and Ontario appear in previous publications, Berger [2], which stress specific model features and results.

3.3 Stochastic MARKAL

Stochastic MARKAL [12, 13] is a recent extension of MARKAL. It explicitly incorporates multiple scenarios, each with a specified probability of occurrence, and minimizes the expected discounted system's cost. It is based on the multi-stage Stochastic Programming paradigm, Dantzig [6], Wets [31]. The main characteristics of this formulation are summarized below:

1. At each period, there are as many replications of the MARKAL variables as there are different scenario realizations. At those periods when there are more than one realization (e.g. at periods 4 to 9 in the example shown in **Figure 1**), each variable set should be considered as a set of *conditional*

variables, i.e. variables representing contingent actions which will be taken only if the corresponding realization prevails.

2. Each set of variables corresponding to a possible scenario realization must satisfy all MARKAL constraints. Therefore, whatever scenario eventually realizes, the corresponding set of variables (strategy) is feasible. The multi-period constraints, such as capacity transfer, cumulative emission and cumulative resource usage, are thus defined in multiple copies, one copy along each path of the event tree (each such path represents one scenario realization). The single period constraints are repeated as many times as there are different realizations at that period, and that number differs with the period.
3. The objective function (expected cost) is equal to the weighted sum of the individual scenario objective functions (costs), the weight being the scenario's probability of occurrence.

The formulation described above has been implemented on the Extended MARKAL model for Québec, Kanudia and Loulou [12]. The model has a user interface, MUSS (**M**ARKAL **U**sers **S**upport **S**ystem, Goldstein [7]), which manages the input data and generates the linear program for the model. The Extended MARKAL uses OMNI for matrix generation. The existing interface (MUSS) has been extended to capture the event tree probabilities and the different levels of end-use demands and GHG emission limits. Extensive modifications have been made in the OMNI code to generate the required stochastic program. A new report writer collects the appropriate variables and compiles the results for each scenario. These developments constitute a user-friendly software, which can be used to quickly make runs with different assumptions and easily analyze the results.

3.4 Implementation of the Minimax Regret Formulation

The implementation of the MMR program described by **(3)** draws heavily from that of Stochastic MARKAL described above. The first two characteristics of the formulation, that is the replication of variables and constraints, remain the same. However, the third characteristic differs: the objective function is replaced by a single variable ϕ, defined in **(3)**, and the first series of constraints in **(3)** are generated via an additional manipulation of the MPS file of the MARKAL program. The objective function values for perfect information runs, M(z), are computed separately (by solving a deterministic MARKAL program), and used in the right-hand sides of these constraints. The FoxPro program used to create the MPS file for **(3)** takes about 10 minutes to transform a 28,000 rows-40,000 column problem, on a Pentium-133 machine. The resulting linear program is solved using CPLEX.

As illustrated by **Figure 1**, we have selected a range of cumulative GHG abatement from 0% to 40% of the observed 1990 emissions. A cumulative abatement target of x% means that, over the 45 year horizon, the average yearly emission must be x% lower than the observed emission in 1990.

We now establish that the range selected range is indeed compatible with the recent Kyoto Protocol, which establishes the Canadian abatement percentage at 6%, but for the average emission in the period 2008 to 2012 only. The Protocol says nothing about emissions at other periods. It is conceivable that abatement beyond 2012 could be set at levels lower or higher than 6% of 1990 emissions. Such uncertainty is not easy to resolve to-day, and depends not only on a better understanding of the seriousness of the impacts of climate change, but also on the dynamics of international negotiations. Furthermore, in addition to the post-2012 uncertainty, it is not even certain that Canada will be reaching the 6% reduction target in 2008-2012. Finally, a 6% reduction target for Canada does not necessarily translate into a 6% reduction for an individual province such as Quebec. Our uncertainty range therefore, although fairly broad, appears by no means to be *too* broad. The weak 0% reduction target corresponds to a situation where there would be new evidence that the climate change is not as severe as predicted to-day, and/or that the effects of climate change are not as dramatic or imminent as thought. At the upper end, the severe 40% reduction target is not really as high as it may appear at first sight. Indeed, it has been established [10] that if a stabilization of the CO2 concentration at twice the pre-industrial level (i.e. at 550 ppmv) is desired, world emissions would have to be abated considerably more severely than at the rate prescribed in the Kyoto Protocol for 2010. The three intermediate targets (10, 20, and 30%) are evenly spread over the range. Note that, in view of the conjecture of section 2.3, the intermediate values are irrelevant in determining the MMR strategy. They are however relevant to compute an optimal strategy under the expected cost criterion.

4. Analysis of results

4.1 Costs and Regrets of the various Strategies

In our study, seven strategies were examined and compared: the Minimax Regret strategy (MMR), the Minimum Expected Cost strategy (MEV), and five short-sighted strategies. The MEV strategy was studied under the Laplace Criterion, which assumes equal 0.2 probability for all five targets, a traditional way reflecting complete uncertainty on the five outcomes. Each short-sighted strategy assumes a fixed deterministic target, until the resolution date, at which time the strategy is altered to optimally follow whichever target actually realizes. We call these short-sighted strategies Perfect Foresight (PF), for lack of a better term. They have been included in the comparison to demonstrate that ignoring uncertainties (i.e. following a PF strategy) is in general not a good idea. Carefully note that a PF strategy is not at all

the same as the perfect information (PI) strategy described further down, although each PF strategy follows one PI branch up to time *t**.

For each combination of the 7 strategies and the 5 possible target realizations, **Table 1** indicates the corresponding cost, as well as the regret. The regret is obtained (as described in section 2) as the difference between the cost of the strategy and that of the best possible strategy under perfect information. **Table 1** also indicates the Maximum Regret, as well as the Expected Cost of each strategy (see the last columns). The cost of a PF strategy was derived by running the model in two steps as follows: we illustrate the steps by computing the real cost of the strategy that follows the optimal 10% reduction, but when the 20% target realizes. *First step*: the regular MARKAL model is run with a 10% deterministic target, and obtains an objective function value of 1250385 M$. *Second step:* the variables for the first three periods are all frozen as per the solution obtained in step 1, and the model is run again with a 20% constraint, to get the value 1255863 M$, which is the real cost of the 10% PF strategy followed when 20% target realizes. The same procedure is applied to all PF strategies and all possible realizations.

Figure 2 shows essentially the same information as **Table 1**, but in the graphical form of cost-emission trade-off curves. Note that the “strategy” PI (also called Ideal Strategy), represented by the dotted line, is actually not a realistic strategy since it assumes that perfect information is available before the first period starts. It is only used as a benchmark to evaluate the hedging strategies. For any given target, the vertical difference in cost between any strategy and PI is precisely the regret attached to the strategy.

From these results, it is evident that none of the PF strategies performs nearly as well as the MMR or the MEV strategies on the minimax regret criterion. The best of the PF strategies is to follow the 30% target until resolution date, with a worst regret of M$ 4635, i.e. M$ 1322 larger than the MMR worst regret. However, the MEV and MMR strategies appear very close to each other (their worst regrets are 3526 and 3311 M$ respectively, and their expected costs are only 22 M$ apart).

4.2 Sensitivity Analysis

In order to verify whether the closeness of the two approaches was accidental or systematic, we conducted two sensitivity analysis experiments. In the first of these, we adopted a coarser range of targets, namely 0%, 20%, and 40% (**Table 2**). Of course, this did not change the results for MMR, in view of the conjecture. However, the maximum regret of MEV increased to 4035 M$, i.e. 724 M$ more than the MMR regret. Conversely, the expected costs of the two strategies remained close to one another, with a difference of only 109 M$. For this case therefore, the MMR approach is superior to the MEV approach in handling uncertainty.

The second sensitivity run consisted in eliminating the target of 40%, keeping only the other four targets (**Table 3**). Again here, the two expected costs remain quite close (their difference is only 26 M$), but the MMR regret is 257 M$ smaller than the MEV regret. We may therefore conclude that MMR is a better approach than MEV in this instance too.

In conclusion, when one combines the above findings with the reduced size of the MMR program (compared to MEV), it is clear that MMR is a very attractive approach to model uncertainty, especially if the number of outcomes of the uncertain event is relatively large. We may add to this that the Minimax Regret criterion seems, at face value, closer to the way in which decision makers would evaluate the risk of making wrong decisions, although we have no formal evidence of this assertion.

4.3 Additional Results on the MMR Hedging Strategy

When facing uncertainty, a good hedging strategy takes into account the important possible outcomes, and strikes an optimal compromise between the negative effects of "guessing wrong". We have seen in the preceding subsection that the MMR strategy succeeds in reducing the worst regret incurred under any outcome of the uncertain target. It is also of interest to examine other dimensions than the cost or the regret of a strategy. For instance, in the case of Greenhouse Gas abatement, it would be important to know what emission trajectory is followed by MMR, as well as the trajectories of some key energy indicators. In this subsection, we restrict our analysis to two examples of such trajectories for MMR, compared to those that would prevail under perfect information. Each of the two figures 3 and 4 illustrates one particular trajectory of the MMR strategy in comparison with the trajectory that would be optimal under perfect information. In order to avoid cluttering the figures, we only show the two extreme branches of the trajectories, i.e. those pertaining to the 0% and the 40% abatement targets.

Figure 3 shows the annual GHG emission under the extreme scenarios with MMR and Perfect Information (PI) strategies. Under the MMR strategy, emissions prior to 2010 adopt a trajectory between the two extreme PI branches, but closer to the 40% branch. In other words, MMR recommends abating emissions significantly as early as in 2005 and 2010, even in the absence of knowledge of the true target. It is therefore not at all a wait-and-see approach. This early abatement approach is, of course, encouraged

by the fact that any abatement occurring in early periods is automatically credited toward the cumulative target when the latter becomes known.

It is interesting to note that the detailed fuel and technology trajectories that result in this emission trajectory are quite diverse: for instance, the trajectory of industrial sector gas consumption (not shown) under the MMR strategy is closer to the 0% abatement perfect foresight. Alcohol consumption in the transport sector (not shown) takes the middle path under MMR. Finally, the trajectory of commercial sector electricity consumption (**Figure 4**) is particularly interesting, as it lies *outside* the limits defined by the two extreme perfect information strategies. This clearly shows that uncertainty affects the competitiveness of energy sector options in ways that are not easily predictable without an explicit, rigorous treatment of uncertainty. The traditional scenario approach is seen to be ineffective. Similar conclusions were reached in previous work by the authors [13].

5. Conclusion

In this paper, we have described a Minimax regret formulation of the energy-environment system, in the context of GHG abatement strategies. The formulation has been applied to the Canadian province of Quebec, via the MARKAL model. The MMR approach has some distinct advantages over the MEV approach: the MMR approach does not require the assignment of probabilities to various outcomes; based on the conjecture presented in section 2.3, one can work with much smaller models with the MMR approach; and comparing the costs of MMR and MEV strategies, the MMR looks more robust than MEV, at least on the examples treated.

The second conclusion is common to most models for decision making under uncertainty, and concerns individual trajectories of various decision variables. It has been confirmed in this paper that hedging trajectories may lies almost anywhere, compared to Perfect Information trajectories. Therefore, uncertainty is seen to affect the competitiveness of energy options in a unique way, which can not be captured through isolated analysis of pure (PI) scenarios. This is a strong reason to adopt formal treatment of uncertainty, over the traditional scenario-by-scenario analysis.

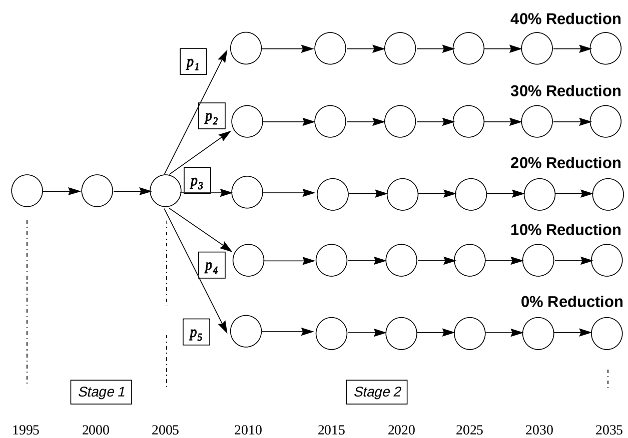

Figure 1 Event Tree for Stochastic MARKAL Example: each realization represents a particular cumulative GHG abatement level.

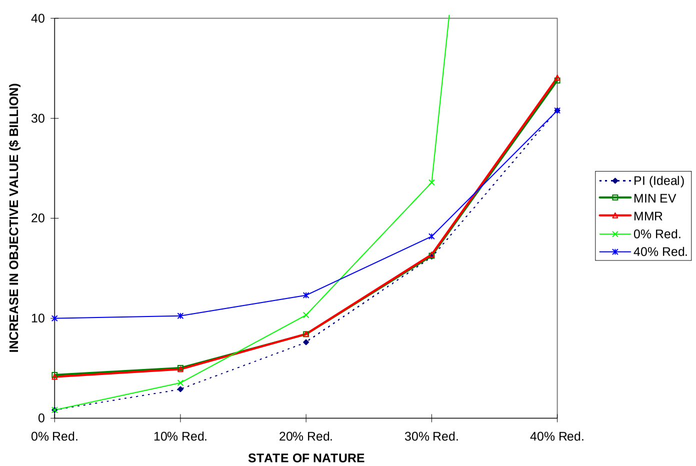

Figure 2 Cost-Emission Trade-off Curves with 0, 10, 20, 30 and 40% Scenarios

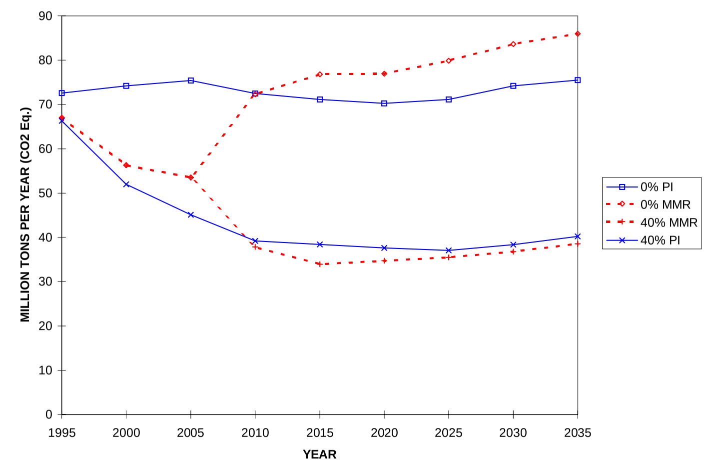

Figure 3 Annual GHG Emission

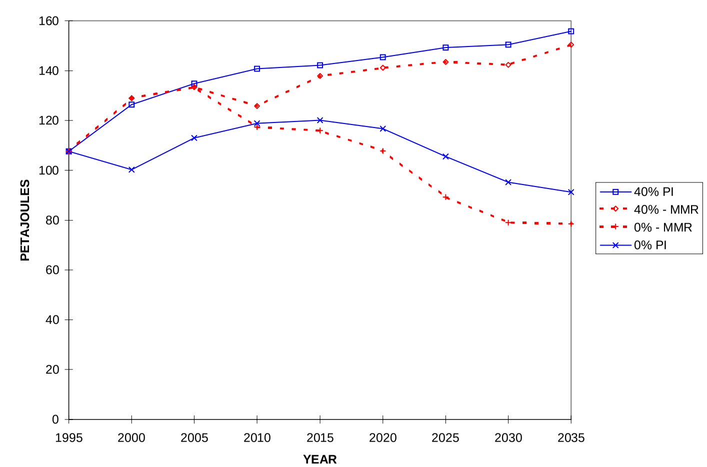

Figure 4 Electricity Consumption in the Commercial Sector

Table 1 Objective Function Values and Regrets with 0, 10, 20, 30 and 40% Scenarios

SYSTEM COST

	SCENARIO REALIZATION					
STRATEGY	**0% Red.**	**10% Red.**	**20% Red.**	**30% Red.**	**40% Red.**	**Expected Value**
PF 0% Red.	1248297	1251024	1257793	1271067	1391320*	1283900
PF 10% Red.	1248611	1250385	1255863	1266496	1340820	1272435
PF 20% Red.	1249674	1250793	1255079	1264183	1296129	1263172
PF 30% Red.	1251599	1252220	1255663	1263651	1282896	1261206
PF 40% Red.	1257480	1257728	1259781	1265676	1278261	1263785
MEV	1251823	1252503	1255890	1263739	1281284	1261048
MMR	1251608	1252395	1255916	1263864	1281569	1261070

***** This value is for 38% reduction. It is *infeasible* to implement a 40% reduction target if the 0% strategy is followed till the year 2010.

REGRETS

	SCENARIO REALIZATION					
STRATEGY	**0% Red.**	**10% Red.**	**20% Red.**	**30% Red.**	**40% Red.**	**Max. Regret**
PF 0% Red.	0	639	2714	7416	113059	113059
PF 10% Red.	314	0	784	2845	62559	62559
PF 20% Red.	1377	408	0	532	17868	17868
PF 30% Red.	3302	1835	584	0	4635	4635
PF 40% Red.	9183	7343	4702	2025	0	9183
MEV	3526	2118	811	88	3023	3526
MMR	3311	2010	837	213	3308	3311

Table 2 Objective Function Values and Regrets with 0, 20 and 40% Scenarios

SYSTEM COST

	SCENARIO REALIZATION			
STRATEGY	**0% Red.**	**20% Red.**	**40% Red.**	**Expected Value**
PF 0% Red.	1248297	1257793	1391320*	1299137
PF 20% Red.	1249674	1255079	1296129	1266961
PF 40% Red.	1257480	1259781	1278261	1265174
MEV	1252332	1256180	1280255	1262922
MMR Regret	1251608	1255916	1281569	1263031

REGRETS

	SCENARIO REALIZATION			
STRATEGY	**0% Red.**	**20% Red.**	**40% Red.**	**Max. Regret**
PF 0% Red.	0	2714	113059	113059
PF 20% Red.	1377	0	17868	17868
PF 40% Red.	9183	4702	0	9183
MEV	4035	1101	1994	4035
MMR Regret	3311	837	3308	3311

Table 3 Objective Function Values and Regrets with 0, 10, 20, and 30% Scenarios

SYSTEM COST

	SCENARIO REALIZATION				
STRATEGY	**0% Red.**	**10% Red.**	**20% Red.**	**30% Red.**	**Expected Value**
PF 0% Red.	1248297	1251024	1257793	1271067	1257045
PF 10% Red.	1248611	1250385	1255863	1266496	1255339
PF 20% Red.	1249674	1250793	1255079	1264183	1254932
PF 30% Red.	1251599	1252220	1255663	1263651	1255783
MEV	1249541	1250721	1255107	1264306	1254919
MMR Regret	1249284	1250616	1255240	1264638	1254945

REGRETS

	SCENARIO REALIZATION				
STRATEGY	**0% Red.**	**10% Red.**	**20% Red.**	**30% Red.**	**MAX. REGRET**
PF 0% Red.	0	639	2714	7416	7416
PF 10% Red.	314	0	784	2845	2845
PF 20% Red.	1377	408	0	532	1377
PF 30% Red.	3302	1835	584	0	3302
MEV	1244	336	28	655	1244
MMR Regret	987	231	161	987	987

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